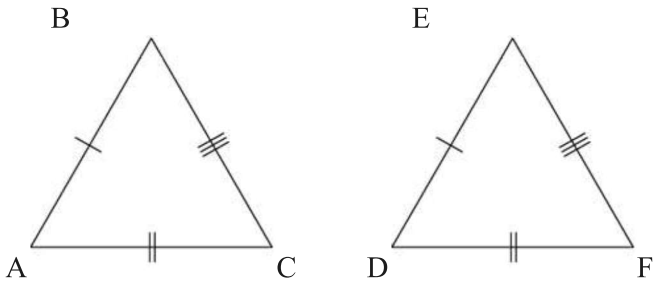
Chapter 6: Congruence and Similarity

Tests of Congruency

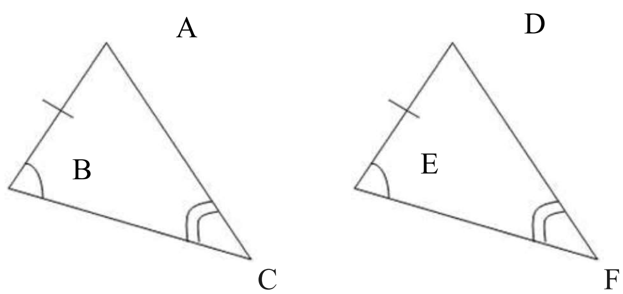
1. Side – Side – Side



∠A = ∠D AB = DE
∠B = ∠E BC = EF
∠C = ∠F AC = DF

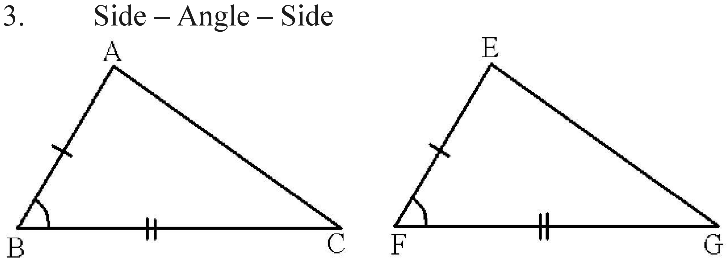
So ΔABC ≅ ΔDEF (SSS Rule)

2. Angle – Side – Angle



Let BC = EF, ∠B = ∠E, AB = DE, ∠C = ∠F
So ΔABC ≅ ΔDEF (ASA Rule)

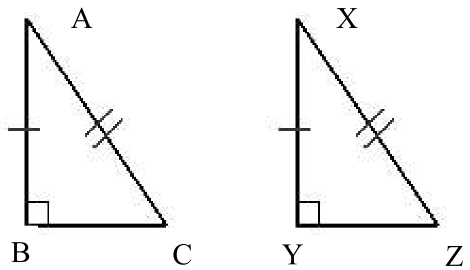
3. Side – Angle – Side



AB = EF
∠ABC = ∠EFG
BC = FG

So ΔABC ≅ ΔEFG (SAS Rule)

4. Right angled – Hypotenuse – Side



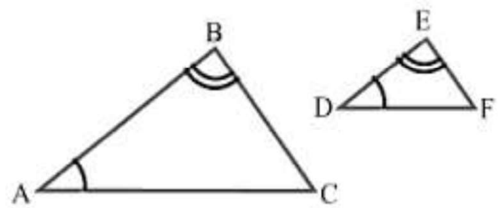
∠B = ∠Y = 90 º
AB = XY
AC = XZ
So ΔABC ≅ ΔXYZ (SAS Rule)

Tests for Similarity

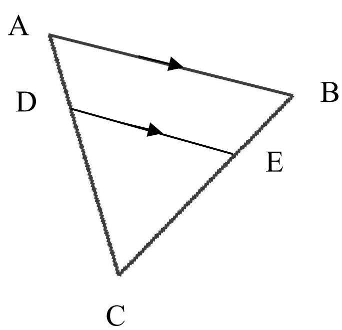
ΔABC and ΔDEF are similar if,

1. 2 pairs of corresponding angles are equal

i.e. ∠A = ∠D, ∠B = ∠E (AA)



Example



In ΔABC and ΔDEC,

∠ACB = ∠DCE (common)

∠ABC = ∠DEC (corresponding angles)

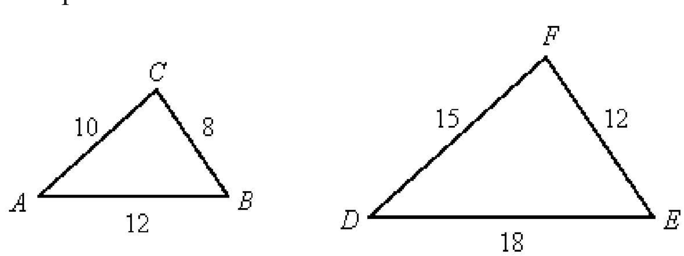
∠BAC = ∠EDC (corresponding angles)

∴ ΔABC is similar to ΔDEC (AA Similarity)

2. 3 pairs of corresponding sides are in the same ratio

i.e. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (SSS)

Example



In ΔABC and ΔDEF,

$\frac{AB}{DE} = \frac{12}{18} = \frac{2}{3}$, $\frac{BC}{EF} = \frac{8}{12} = \frac{2}{3}$, $\frac{AC}{DF} = \frac{10}{15} = \frac{2}{3}$

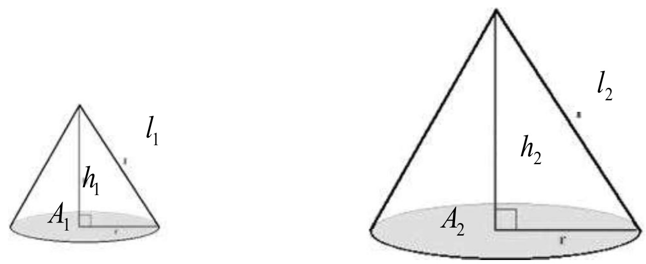
∴ ΔABC is similar to ΔDEF (SSS Similarity)

Ratio of Areas of Similar Figures

If 2 figures are similar, then the ratio of their areas is given by

$$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{l_1}{l_2}\right)^2$$

Ratio of Volumes of Similar Solids



For 2 geometrically similar solids,

the ratio of their length or heights are $l_1 : l_2$ and $h_1 : h_2$

then the ratio of their areas is given by $\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{l_1}{l_2}\right)^2$

and the ratio of their volumes is given by $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 = \left(\frac{l_1}{l_2}\right)^3$